Quantifying (net) Energy and Complexity of the U.S. Economy

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Carey W. King
Question:

Can we find a link among Energy and Modern Economic Structure?

Data: U.S. Input-Output tables from Bureau of Economic Analysis
I use a combination of ideas to relate energy and economic structure

Howard T. Odum (systems ecology)

Charles Hall (ecology, net energy analysis)

Joseph A. Tainter (anthropology)

Robert E. Ulanowicz (systems ecology, information theory)

Me today (engineer, systems)
Tainter: energy-complexity spiral

• Increased complexity needs resources
  – “… most of the time complexity increases to solve problems.”
  – Societies “… subsequently must produce more energy and other resources to pay for the increased complexity.”


• **Theory:** Increasing gross &/or net energy resources facilitate increased complexity

• **This presentation:** quantify “structure” (~ complexity) of modern U.S. economy
C. Hall: EROI = energy return on energy invested (measure of net energy)

- How much energy to needed to obtain energy
- Concept is simple

\[
\text{EROI} = \frac{\int \text{(Power Output)} \, dt}{\int \text{(Power invested)} \, dt}
\]

- I use an interpretation with annual economic data (NPR=net power ratio; see King et al., 2015, Energies):

\[
\text{NPR}_{\text{econ}} = \frac{\$ \text{ out}}{\$ \text{ invested for energy}} = \frac{\text{GDP}}{\$ \text{ yr spent by energy & food sectors}}
\]
Ulanowicz: Information theory metrics to quantify distribution of network flows

- **X**, Mutual Constraint
  - Increases with concentration of flows, “efficiency”

- **Ψ**, Conditional Entropy
  - Increases with more dispersed and uniform flows, “overhead” or “potential reserve”

- **H** = **X** + **Ψ**, Information Entropy
  - Total structural “capacity” (more enables more adaptation)

\[
X = \sum_{i,j} \frac{T_{ij}}{T_{..}} \ln \left( \frac{T_{ij} T_{..}}{T_{i..} T_{..j}} \right)
\]

\[
\Psi = -\sum_{i,j} \frac{T_{ij}}{T_{..}} \ln \left( \frac{T_{ij}^2}{T_{i..} T_{..j}} \right)
\]

\[
H = -\sum_{i,j} \frac{T_{ij}}{T_{..}} \ln \left( \frac{T_{ij}}{T_{..}} \right)
\]

\(T_{..} = \text{total flow in system}\)

Ulanowicz (2009)
Understanding Ulanowicz’s information theory interpretation for networks

- **Uniform (fully connected)**
  - $\chi$ (Mutual Constraint) @ minimum (=0)
  - $\Psi$ ( Conditional Entropy) @ maximum
  - “redundant” undifferentiated organization

- **One pathway of equal flows**
  - $\chi$ (Mutual Constraint) @ maximum
  - $\Psi$ ( Conditional Entropy) @ minimum (=0)
  - “efficient” differentiated organization
There is a defined information theory ‘phase space’ for networks

- Point of maximum conditional entropy
- Maximum connectivity
- Not complex (no distinct parts)
- No hierarchy

\[ \ln(N^2) \]

\[ \ln(N) \]

Mutual Constraint (X)

Conditional Entropy (Ψ)

\[
\begin{array}{cccc}
1/4 & 1/4 & 1/4 & 1/4 \\
1/4 & 1/4 & 1/4 & 1/4 \\
1/4 & 1/4 & 1/4 & 1/4 \\
1/4 & 1/4 & 1/4 & 1/4 \\
\end{array}
\]

\[
\begin{array}{cccc}
1 & & & \\
& 1 & & \\
& & 1 & \\
& & & 1 \\
\end{array}
\]
There is a defined information theory ‘phase space’ for networks

- Point of maximum conditional entropy
  - Maximum connectivity
  - Not complex (no distinct parts)
  - No hierarchy

- Point of maximum mutual constraint
  - Maximum effective number of roles
  - Not resilient (no redundancy)
  - No hierarchy
There is a defined information theory ‘phase space’ for networks

- \( \ln(N^2) \)
- \( \ln(N) \)
- Conditional Entropy (Ψ)
- Mutual Constraint (X)

- No network
- Full “hierarchy” (Gini = 1)
There is a defined information theory ‘phase space’ for networks
Short description of U.S. Input-Output data

• U.S. input-output (Use) summary benchmark tables 1947 to 2012
  – 79 sectors 1947-1992 (SIC format)
  – 69 sectors 1997-2012 (NAICS format)

• “Harmonized” to a consistent 37-sector description across all years
  – There are 5 “energy and food” sectors
    • Important for relating to net energy concept
Where do the U.S. Input-Output (Use) Tables reside?

- Shown:
  - Intermediate transactions only
  - No GDP (output) or Value Added (input)
Results vs. time

- Information Theory Metric
- Closed: intermediate I-O only
- Information Entropy (H)
- Conditional Entropy (Psi)
- Mutual Constraint (X)

Graph showing time-wise variation in different information theory metrics from 1945 to 2015.
Results vs. Gross Power (total primary energy consumption per year)
Results vs. Net Power

Phase 1:
(1947-1967)

<table>
<thead>
<tr>
<th>Metric</th>
<th>Change (%/yr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gross Power</td>
<td>+4.1</td>
</tr>
<tr>
<td>Net Power Ratio</td>
<td>+2.7</td>
</tr>
<tr>
<td>H (info. Entropy)</td>
<td>+0.4</td>
</tr>
<tr>
<td>Ψ (cond. Entropy)</td>
<td>+0.6</td>
</tr>
<tr>
<td>X (mutual const.)</td>
<td>-0.3</td>
</tr>
</tbody>
</table>
## Results vs. Net Power

### Phase 2:
(1967-2002)

<table>
<thead>
<tr>
<th>Metric</th>
<th>Change (%/yr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gross Power</td>
<td>+1.9</td>
</tr>
<tr>
<td>Net Power Ratio</td>
<td>+1.3</td>
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<tr>
<td>H (info. Entropy)</td>
<td>-0.1</td>
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<tr>
<td>Ψ (cond. Entropy)</td>
<td>+0.1</td>
</tr>
<tr>
<td>X (mutual const.)</td>
<td>-0.8</td>
</tr>
</tbody>
</table>

![Graph showing changes in various metrics over time](image_url)
Results vs. Net Power

Phase 3: (2002-2012)

<table>
<thead>
<tr>
<th>Metric</th>
<th>Change (%/yr)</th>
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</thead>
<tbody>
<tr>
<td>Gross Power</td>
<td>-0.3</td>
</tr>
<tr>
<td>Net Power Ratio</td>
<td>-2.1</td>
</tr>
<tr>
<td>$H$ (info. Entropy)</td>
<td>-0.4</td>
</tr>
<tr>
<td>$\Psi$ (cond. Entropy)</td>
<td>-0.6</td>
</tr>
<tr>
<td>$X$ (mutual const.)</td>
<td>+1.2</td>
</tr>
</tbody>
</table>
### Summary of Trends

<table>
<thead>
<tr>
<th></th>
<th>Gross Power</th>
<th>Net Power (economic)</th>
<th>Conditional Entropy</th>
<th>Mutual Constraint</th>
</tr>
</thead>
<tbody>
<tr>
<td>Phase 1 (1947-1967)</td>
<td>↑</td>
<td>↑</td>
<td>↑</td>
<td>↓</td>
</tr>
<tr>
<td>Phase 2 (1967-2002)</td>
<td>↑</td>
<td>↑</td>
<td>↔</td>
<td>↓</td>
</tr>
<tr>
<td>Phase 3 (2002-2012)</td>
<td>↓</td>
<td>↓</td>
<td>↓</td>
<td>↑</td>
</tr>
</tbody>
</table>

![Graph showing trends over different phases](image_url)
Takeaways

• There is a connection between the structure of the (U.S.) economy and net energy
  – Economy started concentrating flows more after “food and energy” became more expensive
• Energy and food have stopped getting cheaper since ~2000
  – Fundamental economic structural change related to energy (food = energy)
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There is a defined information theory ‘phase space’ for networks.

\[ \ln(N^2) \]

Line of maximum information entropy
\[ (H = X + \Psi) \text{ at any given } X \text{ (or } \Psi) \]

\[ \ln(N) \]

Mutual Constraint (X)

Conditional Entropy (Ψ)
There is a defined information theory ‘phase space’ for networks

Line of maximum information entropy
\( H = X + \Psi \) at any given \( X \) (or \( \Psi \))

There is no known “optimal” balance point between efficiency (\( X \)) and redundancy (\( \Psi \))
How do metrics change as you simplify the model?

\[
\text{Info. Entropy} = \text{Mutual Constraint} + \text{Cond. Entropy} \\
(H = X + \Psi)
\]

**Table 3** Table showing correlations for 37, 12, 7, and each 2-sector breakdown for information entropy for the closed model without Value Added as inputs and GDP as outputs.

<table>
<thead>
<tr>
<th>37</th>
<th>12</th>
<th>7</th>
<th>2</th>
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<tr>
<td>1</td>
<td>0.46</td>
<td>0.14</td>
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<tr>
<td>-</td>
<td>1</td>
<td>0.94</td>
<td>-0.76</td>
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<tr>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-0.83</td>
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<tr>
<td>-</td>
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<td>1</td>
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</tbody>
</table>

King, C.W. (in review)
How do metrics change as you simplify the model?

Info. Entropy = Mutual Constraint + Cond. Entropy

\( H = X + \Psi \)

King, C.W. (in review)
How do metrics change as you simplify the model?

Info. Entropy = Mutual Constraint + Cond. Entropy
\( H = X + \Psi \)

Table 7 Table showing correlations for 37, 12, 7, and each 2-sector breakdown for conditional entropy for the closed model without Value Added as inputs and GDP as outputs.

<table>
<thead>
<tr>
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<tbody>
<tr>
<td>1</td>
<td>0.89</td>
<td>0.56</td>
<td>-0.1</td>
<td>0.65</td>
<td>0.48</td>
<td>-0.3</td>
<td>0.7</td>
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<td>-</td>
<td>1</td>
<td>0.82</td>
<td>-0.23</td>
<td>0.61</td>
<td>0.79</td>
<td>-0.69</td>
<td>0.9</td>
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<td>-</td>
<td>-</td>
<td>0.86</td>
<td>-0.22</td>
<td>0.57</td>
<td>0.88</td>
<td>-0.8</td>
<td>0.95</td>
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<td>-0.38</td>
<td>-0.2</td>
<td>-0.77</td>
<td>-0.8</td>
<td>0.6</td>
<td>-0.84</td>
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<td>-</td>
<td>-</td>
<td>-0.62</td>
<td>0.13</td>
<td>0.72</td>
<td>-0.74</td>
<td>0.72</td>
<td>0.012</td>
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<td>-</td>
<td>-</td>
<td>1</td>
<td>0.4</td>
<td>-0.1</td>
<td>0.21</td>
<td>0.64</td>
<td>0.93</td>
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<td>0.52</td>
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</tr>
</tbody>
</table>

King, C.W. (in review)
hierarchy = \ln(N^2) - \sqrt{X^2 + \Psi^2}

hierarchy normalized = 1 - \frac{\sqrt{X^2 + \Psi^2}}{\ln(N^2)}
“Hierarchy” started increasing from 1997 to 2002

1997 had fewer intermediate energy expenditures

2002 had fewest intermediate “food and energy” expenditures
Quantifying “complexity” as a single number is problematic ...

\[ \ln(N^2) \]

\[ \ln(N) \]

Mutual Constraint (X)

Conditional Entropy (Ψ)

Not Complex
Quantifying “complexity” as a single number is problematic ...
Quantifying “complexity” as a single number is problematic ...

\[
\ln(N^2) \quad \ln(N) \quad \text{Mutual Constraint (X)} \quad \text{Conditional Entropy (Ψ)}
\]

\[ (X_0, Ψ_0)_1 \quad (X_0, Ψ_0)_2 \quad (X_0, Ψ_0)_3 \quad (X_0, Ψ_0) \ldots \]
Quantifying “complexity” as a single number is problematic ...

\[ \text{complexity} = \sqrt{\ln(N^2) - 2X_o}^2 + X_o^2 - \sqrt{(\Psi - (\ln(N^2) - 2X_o))^2 + (X - X_o)^2} \]

\[ \text{complexity normalized} = \frac{\text{complexity}}{\text{maximum complexity}} = 1 - \frac{\sqrt{(\Psi - (\ln(N^2) - 2X_o))^2 + (X - X_o)^2}}{\sqrt{\ln(N^2) - 2X_o}^2 + X_o^2} \]
Quantifying “complexity” as a single number is problematic ...

\[ \ln(N^2) \]

\[ \ln(N) \]

Mutual Constraint (X)

Conditional Entropy (Ψ)

\[ a_0 = 0.25 \]

\[ a_0 = 0.33 \]

\[ a_0 = 0.43 \]

\[ a_0 = 0.54 \]

“Optimal point”:

\[ a_0 = \frac{X_0}{X_0 + \psi_0} \]
Quantifying “complexity” is problematic
Results vs. Net Power

- Net Power Ratio = GDP / (food and energy sector intermediate spending)
When you consider inputs (Value Added) and outputs (GDP) the trends are a little different.
Where do the U.S. Input-Output (Use) Tables reside?

- Shown:
  - Intermediate transactions with GDP (outputs) and Value Added (inputs)
Results vs. time
Results vs. time

- Information Entropy (H)
- Conditional Entropy (Psi)
- Mutual Constraint (X)
- Closed: intermediate I-O only
Results vs. Net Power