Introduction to

Calculus based (Bio-)Physical Economics

Synthesis of Social and Natural Sciences?

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Calculus
Macroeconomics
Production
Growth
Calculus
Question: Every investor would like to calculate the course of profits or share prices in advance (ex ante) like the flight of a ball or an arrow! Why is this not possible?

Answer: All economic terms may be divided into two categories: they may be calculated before or afterwards: \textit{ex ante} (at all times certain) or \textit{ex post} (in the beginning uncertain)
Neoclassical Macroeconomics

Solow model of economic growth:

\[ Y = F(K, L) \]

**But:** Income \((Y)\) is an *ex post* term (uncertain).
A function \((F)\) is *ex ante* term (certain)

*Uncertain* cannot be equal to *certain!*

\[ Y \neq F(K, L) \]
Solution:

Calculus of two variables, because economics depends on two production factors: capital and labor.

1. **Riemann** integral of exact differential \( d\, F \)

   \[
   \int d\, F = F(B) - F(A) \quad \text{depends on limits A and B} \\
   \text{path independent (certain)}
   \]

2. **Stokes** integral of not exact differential \( \delta\, Y \)

   \[
   \int_p \delta\, Y = Y_p(K, L) \quad \text{path (p) dependent integral} \\
   \text{there is no stem function } Y! \quad \text{(uncertain)}
   \]

3. **Integrating** factor \( \lambda \)

   \[
   \delta\, Y = \lambda\ d\, F(K, L) \quad \text{replaces } Y \neq F(K, L)
   \]
Closed integrals

Closed Riemann integrals of exact differentials are always zero!

(Path independent integral, certain)
Closed integrals

Closed Stokes integrals of not exact differentials are never zero!

(Path dependent integral, uncertain)
Examples for closed Stokes integrals

A household (H) works in industry (In) earning 100 € per day and spends 90 € for food and goods. The surplus is 10 € per day.

\[ \delta M = Y_H - C_H = \Delta M_H \]
Examples for closed Stokes integrals

The closed Stokes integral is a basis of Bio-Physical-Economics

- **Blood circuit**
  - $C + O_2$
  - $C O_2$
  - Biology

- **Heat pump**
  - Physics

- **Production circuit**
  - Economics

Exemplary processes:
- **Blood circuit**: Blood flow, oxygenation
- **Heat pump**: Thermal energy transfer
- **Production circuit**: Economic production cycle
Macroeconomics
Double-entry accounting: Excel balance

A household (H) works in industry (In) earning 100 € per day and spends 90 € for food and goods. The surplus is 10 € per day.

\[
\begin{align*}
Y_H &= 100 \text{ €} \\
C_H &= -90 \text{ €} \\
\Delta M_H &= 10 \text{ €} \\
W_H &= -100 \text{ €} \\
G_H &= 90 \text{ €} \\
\Delta P_H &= -10 \text{ €}
\end{align*}
\]

(monetary units: €) \hspace{1cm} (energy units, kcal, MJ)

monetary account \hspace{1cm} + \hspace{1cm} productive account \hspace{1cm} = \hspace{1cm} 0

Double-entry accounting – the basis of economics
Double-entry accounting: Stokes integral

A household (H) works in industry (In) earning 100 € per day and spends 90 € for food and goods. The surplus is 10 € per day.

\[ \oint \delta M + \oint \delta L = 0 \]

The monetary account measures the productive account (in €)

*This is the fundamental integral of economics*
The fundamental laws of Economics

\[ \oint \delta M = - \oint \delta L \]  

monetary units  

Fundamental law
The fundamental laws of Economics

monetary units

\[ \oint \delta M = - \oint \delta L \quad \text{Fundamental law} \]

1. law: \( \delta M = dK - \delta L \quad \text{Diff. double entry balance} \)

\text{Profit} = \text{Capital} \quad \text{Labour}
The fundamental laws of Economics

monetary units

\[ \int \delta M = - \int \delta L \]

Fundamental law

1. law: \( \delta M = d K - \delta L \)

Diff. double entry balance

Profit = Capital \hspace{1mm} Labour

2. law: \( \delta M = \lambda \hspace{1mm} d F \)

Existence proof of the production function \( F \)
The fundamental laws of Economics

monetary units

$$\int \delta M = - \int \delta L$$

1. law: \[ \delta M = d K - \delta L \]

2. law: \[ \delta M = \lambda \ d F \]
**The fundamental laws of Economics**

- **monetary units**
  \[ \int \delta M = - \int \delta L \]

- **energy units**
  \[ \int \delta Q = - \int \delta W \]

1. **law:** \( \delta M = d K - \delta L \)

2. **law:** \( \delta M = \lambda \ d F \)

Oilprice: \( \rightarrow \) 118 MJ = 33 kWh
The fundamental laws of Economics

<table>
<thead>
<tr>
<th>Econo</th>
<th>-</th>
<th>Physics</th>
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</thead>
<tbody>
<tr>
<td>$\delta M$ surplus</td>
<td>$\leftrightarrow$</td>
<td>$\delta Q$ heat  (uncertain)</td>
</tr>
<tr>
<td>$d K$ capital</td>
<td>$\leftrightarrow$</td>
<td>$d E$ energy  (certain)</td>
</tr>
<tr>
<td>$\delta L$ labour</td>
<td>$\leftrightarrow$</td>
<td>$\delta W$ work  (uncertain)</td>
</tr>
<tr>
<td>$d F$ prod. function</td>
<td>$\leftrightarrow$</td>
<td>$d S$ entropy  (certain)</td>
</tr>
<tr>
<td>$\lambda$ living standard</td>
<td>$\leftrightarrow$</td>
<td>$T$ temperature</td>
</tr>
</tbody>
</table>

Unit: €,$, ¥, £ $\leftrightarrow$ Unit: J, kWh, kcal

dimension: money $\leftrightarrow$ dimension: energy
The temperature ($\lambda$) of an economic/social system:

GDP per capita and energy consumption per capita
Production
Double-entry accounting: Stokes integral

A household (H) works in industry (In) earning 100 € per day and spends 90 € for food and goods. The surplus is 10 € per day.

(monetary units: €)    (energy units, kcal, MJ)

\[ \oint \delta M + \oint \delta L = 0 \]

The monetary account measures the productive account (in €)

This is the fundamental integral of economics
The mechanism of production (Capitalism)

\[ \oint \delta M = - \oint \delta L \]
Closed Stokes integrals lead to a two-level Carnot process:

**Production process:** two separate levels $\lambda_1$ and $\lambda_2$:
- Companies: capital and labor
- Trade: buyer and seller
- Banks: investors and savers
- Societies: rich and poor
- Economies: 1. world and 2. world

**Carnot process:** two separate temperatures $T_1$ and $T_2$:
- Motors: hot and cold
- Refrigerators: hot and cold
- Heat pumps: hot and cold
- Generators: hot and cold

**Econophysics:** Production is a Carnot process
- Motor, generator $\rightarrow$ productive circuit, company
- Heat pump, fridge $\rightarrow$ monetary circuit, bank
Growth
Economic growth is a two level process

Growth by international trade \((p < 0.5)\)

US - China 1990-2010
Economic growth by trade in the world

Growth in the world: Labour: 3 %  Capital 6 %

Data by Thomas Piketty
Linking Social Sciences to Natural Science and Engineering

Economics
1. Production and trade (Carnot process)
2. Economic growth and degrowth (heat of motors)
3. Income: How much should a manager earn? (barometer)
4. Financial markets, financial crisis (statistics, roulette)

Societies
1. Homogeneous societies: hierarchy (collective), democracy (individual) and global state (solid, liquid, gas)
2. Heterogeneous societies: Cooperation, integration, segregation and aggression: Example: US integration, The war in Bosnia (binary alloys)
3. Public decisions: The analysis of US elections (crystal growth)
4. Mobility in Germany (diffusion)

Politics
1. Afghanistan, China (solids)
2. Standard of living and politics: the Arab revolution (phase diagram)
3. From Marshall plan to present EU problems: (conduction of heat)
4. Migration and refugies in a globalized world (atomic diffusion)
Thank you for your attention
Economic growth

\[ d Y_1(t) = p (Y_2 - Y_1) d \alpha t \]
\[ d Y_2(t) = (1 - p) (Y_2 - Y_1) d \alpha t \]

\[ p \quad : \quad \text{distribution of surplus to the lower side} \]

Two interdependent economic systems

Two interdependent countries

\[ 0 < p \leq 0.5 \]

\( p = 0.75 \)
Economic growth is a two level process

States between Capitalism and Communism

**Efficiency:** $\eta = (\lambda_2 - \lambda_1) / \lambda_2$

$p = 0.00$: Colonialism: $\eta \to \text{Max!}$ (Very high prices, very low wages)  
strong market (Colonal Africa)

$p = 0.10$: Capitalistic: $\eta \to \text{Max!}$ (High prices, low wages)  
strong market (USA, China, Germany)

$p = 0.25$: Socialism: $\eta \to \text{Min!}$ (Lower prices, higher wages)  
weak market (France)

$p = 0.50$: Fair deal: $\eta \to \text{const.}$ (Prices and wages nearly equal)  
nearly equal)

$p = 0.75$: Stagnation: $\eta \to \text{Decreasing!}$ (Low prices, too high wages)  
weakening market (Greece)

$p = 1.00$: Communism: $\eta \to \text{zero!}$ (One class system)  
the market will break down (USSR, DDR)

Communism is like opening the door of a fridge mixing hot and cold.  
The refrigerator will stop.
Economic growth and fertility in the 90 biggest countries

GdP per capita and fertility for 90 countries
The fundamental laws of BioEconoPhysics

\[ \oint \delta Q = - \oint \delta W \]

\[ \delta W = d E - T d S \]

<table>
<thead>
<tr>
<th>Bio</th>
<th>-</th>
<th>Econo</th>
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<th>Physics</th>
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</thead>
<tbody>
<tr>
<td>(\delta Q) ↔ life</td>
<td>(\delta M) ↔ surplus</td>
<td>(\delta Q) ↔ heat</td>
<td></td>
<td></td>
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<tr>
<td>(d E) ↔ food</td>
<td>(d K) ↔ capital</td>
<td>(d E) ↔ energy</td>
<td></td>
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<tr>
<td>(\delta W) ↔ work</td>
<td>(\delta P) ↔ production</td>
<td>(\delta W) ↔ work</td>
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<td></td>
</tr>
<tr>
<td>(d S) ↔ genetics</td>
<td>(d F) ↔ prod. Function</td>
<td>(d S) ↔ entropy</td>
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</tr>
<tr>
<td>(\Lambda) ↔ food level</td>
<td>(\lambda) ↔ living standard</td>
<td>(T) ↔ temperature</td>
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**Stokes integral:** ↔ **Mechanism of production**

Animals: Blood circuit
Plants: Chlorohyll circuit
Production circuit
Carnot motor
Outlook

Social sciences ↔ Natural sciences

Accounting
ex ante, ex post
accounts (balanced, unbalanced)
↓
Macro-economics
capital, labour
↓
Business
capitalism: companies, banks
↓
Micro-economics
Lagrange
↓
Finance
capital, finance
↓
Social Sciences
Homo- and heterogen agents
networks
↓
Politics
hierarchy, democracy, global

↔

Calculus
differentials (exact, not exact)
↔
integrals (Riemann, Stokes)
↓
Thermodynamics
energy, work
↓
Mechanical engineering
Carnot: motors, heat pumps
↓
Statistical mechanics
free energy
↓
Statistics
energy, entropy
↓
Chemistry
atoms, molecules
↔
crystal structures
↓
Materials
solid, liquid, gas
<table>
<thead>
<tr>
<th>Life sciences</th>
<th>Social sciences</th>
<th>↔</th>
<th>Natural sciences</th>
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<td>Chemistry</td>
<td>Accounting</td>
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<td>Calculus</td>
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<tr>
<td>atoms, ions</td>
<td>ex ante, ex post</td>
<td>↔</td>
<td>differentials (exact, not exact)</td>
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<td>reactions, exo-, endotherm</td>
<td>accounts (balanced, unbalanced)</td>
<td>↔</td>
<td>integrals (Riemann, Stokes)</td>
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<tr>
<td>Biology</td>
<td>Macro-economics</td>
<td>↔</td>
<td>Thermodynamics</td>
</tr>
<tr>
<td>food, work</td>
<td>capital, labour</td>
<td>↔</td>
<td>energy, work</td>
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<tr>
<td>Biological engineering</td>
<td>Business</td>
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<td>Mechanical engineering</td>
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<td>Blood circuit, Chlorophyll cycle</td>
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<td>Carnot: motors, heat pumps</td>
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<td>Bio-Statistics</td>
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<td>Statistics</td>
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<td>genetics</td>
<td>Finance of capital</td>
<td>↔</td>
<td>energy and entropy</td>
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<tr>
<td>Physio-Chemistry</td>
<td>Social Sciences</td>
<td>↔</td>
<td>Chemistry</td>
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<td>cells</td>
<td>Homo- and heterogen agents</td>
<td>↔</td>
<td>atoms, molecules</td>
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<tr>
<td>cell structures</td>
<td>networks</td>
<td>↔</td>
<td>crystal structures</td>
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<tr>
<td>Life Materials</td>
<td>Politics</td>
<td>↔</td>
<td>Materials</td>
</tr>
<tr>
<td>life in solids, water, air</td>
<td>hierarchy, democracy, global</td>
<td>↔</td>
<td>solid, liquid, gas</td>
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\[ \delta L = dE - \delta W = \Lambda \, dS \quad \delta M = dK - \delta W = \lambda \, dF \quad \delta Q = dE - \delta W = T \, dS \]
<table>
<thead>
<tr>
<th><strong>Linking Social Sciences to Natural Science and Engineering</strong></th>
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<td><strong>Accounting</strong></td>
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<tr>
<td>balances</td>
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<td><strong>Macro-economics</strong></td>
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<tr>
<td>income, labour</td>
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<tr>
<td><strong>Business</strong></td>
</tr>
<tr>
<td>companies, banks</td>
</tr>
<tr>
<td><strong>Micro-economics</strong></td>
</tr>
<tr>
<td>capital, finance</td>
</tr>
<tr>
<td><strong>Social Sciences</strong></td>
</tr>
<tr>
<td>collective, individual, global</td>
</tr>
<tr>
<td>crisis, revolution</td>
</tr>
<tr>
<td>cooperation, integration</td>
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<tr>
<td>segregation, aggression</td>
</tr>
<tr>
<td><strong>Politics</strong></td>
</tr>
<tr>
<td>hierachy, democracy, global</td>
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</tbody>
</table>
Unemployment:

The same Carnot mechanism for people and machines

People are replaced by machines

\[ \delta P = dK - \lambda dF \]
\[ \delta W = dE - TdS \]

German people working in agriculture  
1900 : 50 %  
2007 : 2 %
Unemployment:

The same Carnot mechanism for people and machines

People are replaced by machines (dE, dK):

\[ \delta W = dE - TdS \]

White collar work (ordering, planing) is replaced by computers (ordinateurs)
The most important resource is our human intelligence.

Julian S. Simon (1932 - 1998)
Government: Mechanics for repairs and accidents

Market rules and enforcement
Anti trust laws
Bank laws

Regulation of markets by taxes:
Low taxes for low risk productive markets
High taxes for high risk financial markets
High taxes for exceeding manager incomes

Support
Welfare, unemployment support
Start up of small business

Mechanics for repairs in accidents
In economic units or systems the law of survival is still valid for the system, but not anymore for every single element!

This is a positive result for the individual weak, for babies in a family, for sick people in a company or jobless in a society.

This is a negative result for each economic system. Families have to work harder for their babies, companies have to make up for the missing work, the society has to build resources for the jobless.

An economic system may manage to survive, if all individuals cooperate, but a system may fail, if the individual do not cooperate:

e.g. divorced families with children, cities with not cooperating unemployed, the EU with not cooperating members.
Cooperative Systems:

Ecosystem

<table>
<thead>
<tr>
<th>Economics</th>
<th>Biology</th>
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<tbody>
<tr>
<td>retailer</td>
<td>dairy farm</td>
</tr>
<tr>
<td>company</td>
<td>cow</td>
</tr>
<tr>
<td>labor</td>
<td>gras</td>
</tr>
</tbody>
</table>

robust systems
Not cooperating (aggressive) Systems:

Finance
hedge fonds
€ - Europe
tax payer

Biology
lion
buffalo
gras

Ecosystem
Sensitive systems
Economic systems
Critical review of classical models

François Quesnay (1694 – 1774), French economist and medical doctor. His book *Tableau éconmique* (Economic Table, 1758) will be the basis of this discussion.

Adam Smith (1723 – 1790), Scottish philosopher of political economy. His classic work: *The Wealth of Nations* (1776) is still valid today.

Karl Marx (1818 – 1883), German economist, sociologist and journalist. Marx's books *The Communist Manifesto* (1848) and *Das Kapital* (1867–1894) are still used.
Calculus of two variables in meteorology

Stokes integrals (spirals) in meteorology

In high and low pressure systems winds go up and down in spirals
Probability with constraints and risk (r) of detection

\[ P = \frac{(1-r)}{[(1-r) + \exp\left(-\frac{Y}{\lambda}\right)]} \]

Probability of theft at risk of detection, BKA 2002

Probability of murder at risk of detection, BKA 2002
The new production function entropy

When John von Neumann at Bell Labs asked Claude Shannon in 1949 how he was getting on with his information theory, Shannon replied*:

The theory was in excellent shape, except that he needed a good name for "missing information". "Why don’t you call it entropy", von Neumann suggested. "In the first place, it already exists in Boltzmann's statistical mechanics, and in the second place, no one understands entropy very well, so in any discussion you will be in a good position”.

Thank you for your attention
Economic growth

Growth: $p = 0.25$ and $0.10$

Interdependent economic systems

$\ln Y$

Economic level $Y(t)$

Capital - labor
BPE 2016 Washington, D.C.
Entropy: the natural production function

**Shannon:** \( d S = - d N \sum (p_i) \ln (p_i) \geq 0 \)

If we open a perfume bottle, the perfume will leave the bottle and it will never return into the bottle. Entropy is a measure of disorder, of distribution.