Paper presented at the

ISEE conference Challenges and Contributions for a Green Economy

16-19 June 2012

HYPERBOLIC DISCOUNTING AND OPTIMAL ROTATION

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Keywords: Hyperbolic discounting; optimal rotation; time inconsistency

Abstract:

The adoption of hyperbolic rather than exponential discounting in long term planning proceeds with some reluctance. A major concern is the risk of time-inconsistent recommendations. This concern is not justified for projects and programmes with a classical cost-benefit profile, but time inconsistency is possible in projects and programmes with other cost-benefit profiles. Investment in plantations and afforestation differs from the classical profile in important respects and has been identified in the literature as prone to time inconsistency when analysed with hyperbolic discounting. The paper shows that such time inconsistency occurs only when a declining rate of return is applied in a static decision rule where discounting is not relevant. If declining discount rates are only used for calculating present values the suspected time inconsistencies do not occur.

Introduction

Exponential discounting has severe shortcomings in analyses and planning with a very long time perspective (say, more than 25-35 years). The method is developed for analysis of undertakings with a nearer time horizon, but analysis of projects and programmes with a much longer time perspective evolves, in particular, related to the long term transformations to a green economy. Hyperbolic discounting is increasingly recognised as a superior alternative (E.g., Weitzman 1998; Li and Lofgren 2000; Weitzman 2001; Newell and Pizer 2003; Newell and Pizer 2004; Gollier and Weitzman 2010).

The difference is – in brief – that exponential discounting assumes a constant discount rate, whereas hyperbolic discounting assumes discount rates that decline over time. Consequently, exponential discounting at even modest rates tends to discount the far future away, whereas hyperbolic discounting at the same level of initial discount rate can provide for a positive present value of amounts in the far future.

There is, however, some reluctance against adopting hyperbolic discounting. The main concern is the risk of *time-inconsistent* economic recommendations. Time-inconsistency can occur because hyperbolic discounting involves, at the time of planning, a lower rate of discount in the far future. As time actually passes the previously distant future comes closer and its discount rate becomes higher. If the planner reassesses the plan later on in the process, there is a risk that exactly the same data and the same methods as applied in the original planning will lead to opposite conclusions when the plan is reassessed later in the process. Obviously, this is not an attractive property of any economic method.

This paper addresses the question of time-inconsistency related to hyperbolic discounting. It takes departure in the class of projects and programmes with a classical cost-benefit profile and how typical plantation investments differ from this case. The application of declining discount rates in the Faustmann formula and the Ohlin condition as well as the suspected time-inconsistency is then explained. The paper then examines the consistency of the theoretical definition and justification of declining discount rates with the application of declining discount rate in the Faustmann formula and the Ohlin condition. Finally, conclusions are drawn as to theoretically consistent applications of hyperbolic discounting in plantation investment analysis.

The classical cost-benefit profile case

The worry about time inconsistency has been shown to be exaggerated for projects and programmes with classical cost-benefit profiles, that is, a finite investment plan followed by a finite or infinite flow of benefits that can be harvested after completion of the investment plan (Hansen 2006).

This is due to the lower value of the *remaining* investment as the investment plan proceeds and the higher *present value* of the future benefit stream as it comes closer in time. The argument is illustrated in Figure 1.

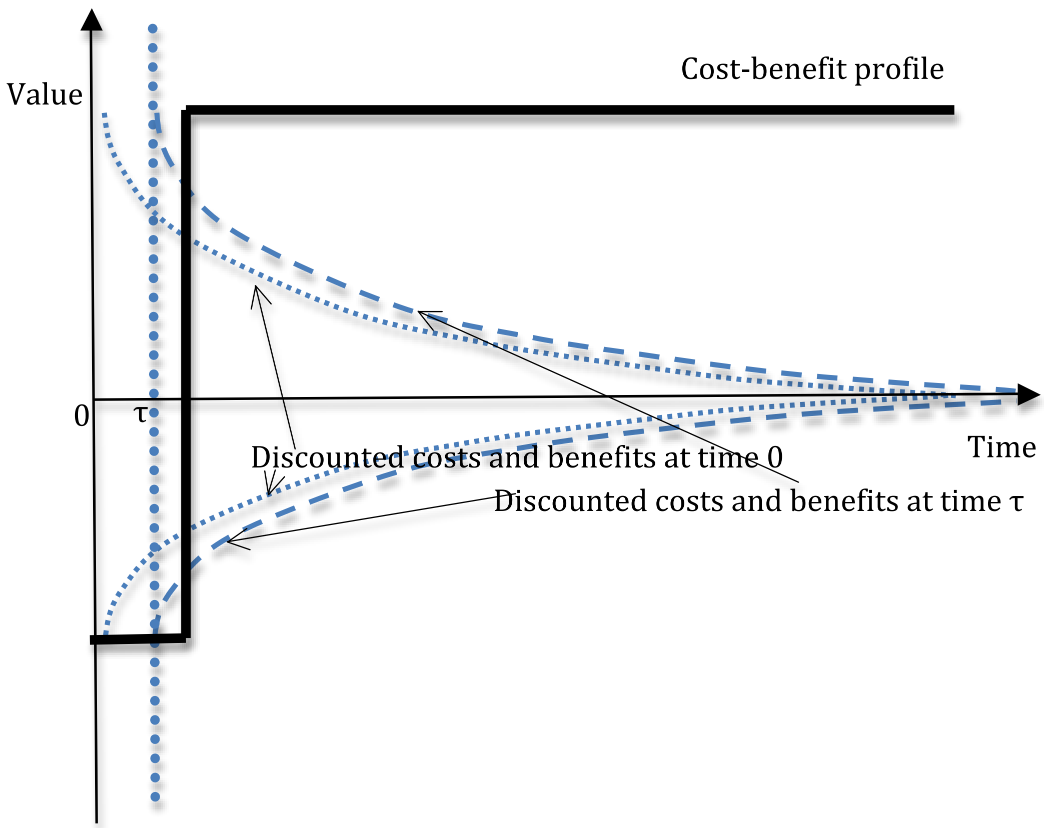

Figure 1. Net present value of a classical cost-benefit profile with reassessment at a later time τ.

Figure 1 shows a project or programme with a classical cost-benefit profile with constant investment costs in an initial phase followed by constant benefits from the point of completion of the planned investments. The net present value at time 0 is the sum of the integral of the discounted investment costs and the integral of the discounted benefits.

If the investments have proceeded according to the plan and we reassess the programme again at time τ, the same logic applies with the dashed curves representing the discounted value of the remaining investment costs at time τ and the discounted value of the expected benefits at time τ.

Note that the area representing the present value of the remaining investments at time τ is identical to the first fraction of the original area, representing the original net present value of the investments at time 0. Thus, the net present value of the costs in such a classical cost-benefit profile will always be smaller at time τ than at time 0 and there is no reason to be worried about time inconsistency in that case.

This property is the same for any discount function – hyperbolic or exponential. Thus, for this class of projects and programmes the discount function in itself does not make an initially recommended plan less recommendable as the planned investments are completed.

Cost-benefit profiles can differ from the classical profile and thus potentially give rise to time inconsistency. E.g., for projects with high decommissioning costs the net present value of the operation phase could become smaller as we approach it. Further discussion of other profiles can be found in Hansen (2006).

Hyperbolic discounting in plantation investment planning

The cost-benefit profile of investments in plantations differs from the classical profile in at least one important respect: The benefits can be cashed in at any time, irrespective of whether the investment plan is completed or not. Concerns have been raised as to the time consistency of economic recommendations of plantation investments if hyperbolic discounting would be used.

In a study on application of hyperbolic discounting in the planning of plantation investments Price (2011) found disturbing time inconsistencies. The optimal rotation period is the growth period from planting to felling that maximises the value of the land when used for forestry. Using a declining rate of discount in the calculation of the optimal rotation period leads to a recommendation of still longer rotation periods the further into the future. When time passes and the future becomes the present, the recommendations would, however, change although nothing else had changed other than the time.

It was, however, also concluded that the *level* of the initial discount rate was more important than whether it was *declining* or not in the far future.

The Faustmann formula and the Ohlin condition

To understand role of discount rates in such forest planning, it is useful to look back at two outstanding contributions to the economics of forestry.

Faustmann (1849) laid the cornerstone of plantation planning with the formula for calculating the wealth of a land area used for forest. He applied two fundamental economic principles: The *net present value* and a *sustainable* production of timber, where "sustainable" refers to a forestry plan with infinitely continuous production in consecutive rotation periods.

Using these principles Faustmann obtained the net present value of foresting the land area decomposed on the value of the bare land and the value of the prospective timber harvest. The mathematical expressions of this below are simplified versions with a more modern notation.

The net present value of a rotation was defined as the discounted value of the prospective net timber rent pS_T at the end of the rotation period T minus the planting cost, k. At the beginning of each rotation period k is invested and every subsequent year the pS_T is "reinvested", that is left on the root, until it is eventually cashed in. Thus, the net present value of an even-aged stand is

(1) $NPV = (pS_TDF_T - k)$

The value of the land designated for forestry, but without the actual timber stock on it is the net present value of the future rents as defined by Faustmann:

(2) $\Pi = (pS_TDF_T - k) + DF_T(pS_TDF_T - k) + DF_{2T}(pS_TDF_T - k) + DF_{3T}(pS_TDF_T - k) + \ldots,$

where DF_T is the discount factor to time T. The formula expresses the net present value of the future timber harvest on a particular piece of land as the discounted net present value (at the time of planting) of each rotation in an infinite sequence of rotations of equal length.

Faustmann applied exponential discounting and reduced by geometric progression (2) to what became known as the Faustmann formula:

(3) $\Pi = (pS_Te^{-iT} - k)(1 - e^{-iT})^{-1},$

where i is the rate of return on alternative investment.

Finally, it is discounted back to the beginning of the period by an exponential discounting discount factor e^{-iT}. The net present value of each rotation is discounted back to the present and the parenthesis $(1 - e^{-iT})^{-1}$ is the geometric progression expression of this.

The question then is, how long a rotation period should be. The standard economist answer would be that it should end when the annual rent equals the alternative returns that you could get by felling the trees, cashing the rent and invest it in something else. Note that the implicit assumption here is that the net rent grows at a declining rate reflecting that the stand has entered the convex growth phase. Moreover, the alternative rate of return is assumed to be constant. Thus, preserving the stand another year would yield less return than terminating the rotation and start a new.

Bertil Ohlin (1921) was probably the first to add that the value of the bare land should be included as well. Thus, we refer to (4) as the Ohlin condition:

(4) $p(dS_T/dT) = ipS_T + i\Pi$

The value of T that maximises Π subject to (4) is the optimal rotation period. It is actually a no-arbitrage condition reflecting an equilibrium state that in principle should be maintained unchanged. However, because of the underlying assumption of a declining growth rate of the timber stock, it implies that $p(dS_T/dT) < ipS_T + i\Pi$ in subsequent years and it is recommendable to convert the timber to cash before that happens.

With a constant *i* – and if everything else is constant - we will at the outset recommend a series of rotations of equal length. If, however, *i* is not constant, but declining over time, formula (4) will lead us to recommend a sequence of still longer rotation periods as illustrated by Colin (2011). When we have done the first rotation and approaches second and third etc., we face higher alternative rates of return than assumed in the original planning of rotations. Reassessing the optimal length of the very same rotations would lead us to recommend shorter periods than we originally planned. And then, maybe, we would not even have recommended the species in question in the first place.

This is exactly the kind of time-inconsistency that should be avoided. Before the application of hyperbolic discounting in forestry planning is dismissed, however, it will be useful to confront the application of declining discount rates in forestry planning with the theoretical background and justification of hyperbolic discounting.

Application of hyperbolic discounting in forestry planning

The Faustmann formula and the Ohlin conditions are quite different properties. The Faustmann formula (3) converts values to present values in order to aggregate them to a net present value. A discount function is obviously indispensible in that operation. The Ohlin condition (4) balances the rate of return on two alternative asset types in the portfolio to check for when it is time for arbitrage. In this operation discounting is not relevant given the assumption that the growth rate of the timber stock will tail off, whereas the rate of return to alternative investments will be constant.

To understand why declining discount rates are only relevant to discounting operations, we have to look at the theoretical argument for declining discount rates. According to the uncertainty-based understanding of hyperbolic discounting as rolled out by Weitzman (1998) the declining discount rate is not a prediction of future rates of interest or other rates of returns to investment should be declining. It is rather a statistical result of the *uncertainty* of future discount rates.

The future can be represented by a number of scenarios, each of which have different, but internally characteristics as to fundamental economic properties including standard rates of return on investment and corresponding discount functions. The discount function of the j^{th} scenario is named a_j and the certainty equivalent discount factor at time *t* is then

(5) $A(t) \equiv \sum p_j \, a_j \,(t)$,

where p_j is the probability that scenario *j* will materialise. The discount factor at a time *t* is not the discount factor derived from the average or expected rate of return on alternative investments:

(6) $\sum p_j \, a_j \,(t) \neq (1-\sum p_j \, i_j)^{-t}$

It is the expected discount *factor*. Whereas we can - and typically will – assume $\sum p_j\, i_j$ constant for each scenario, the corresponding *A(t)* has an instantaneous discount rate that is declining over time. The declining rate of discount is the instantaneous discount rate of $A\ (= A(t+1)/A(t) - 1)$ – not the expected rate of return.

This means that when it comes to the decision rule in (4), the appropriate alternative rate to assume is the expected rate of return on alternative investment such as a weighted average of real interest rates on risk free government bonds.

There are, however, arguments for assuming that the rate of return on investment will decline continuously and approach 0 asymptotically in the far future. They include a.o. the downward trend in low-risk United States government bond real interest rates through the recent two centuries shown by Newel and Pizer (2003). An autoregression analysis extrapolated this trend to a zero rate, but was eventually dismissed in favour of a random walk or mean reverting model. A continuously declining rate of return to investment would also be consistent with the standard assumption of marginally declining productivity in neo-classical growth theory and the high growth rates of developing economies reflecting the transformation from the traditional to the modern economy that at some point will be completed.

These grounds are not sufficiently compelling to predict a continuously declining return to investment in the far future, but they suffice to establish the possibility of a discount rate that asymptotically approaches zero. Thus, the decline of discount rates follows directly of and reflects the *uncertainty* of future growth and rates of return on investment.

This Weitzmanian understanding of hyperbolic discount rates was eventually shared by a range subsequent contributions although they devised very different empirical strategies to estimates the p_js and the a_js in (5) (Li and Lofgren 2000; E.g., Newell and Pizer 2004; Weitzman 2010).

In formula (3), however, hyperbolic discounting is very relevant and hyperbolic discount factors can simply replace exponential discount factors in formula (2).

The impact of this depends on the actual design of the discount factor, which in turns depends on the empirical approach chosen for assumptions of possible discount rates and their probabilities. Generally, however, it must be expected that investment in species that are slow growing and where the benefits materialises further into the future will get a higher net present value. This question is important in forestry since slow growing species and their longer maturation are associated with larger environmental benefits such as biodiversity and aesthetic qualities than fast growing species and their short rotations. The value of the infinitely provided non-timber ecosystem services will also contribute to a higher net present value.

However, in many cases – and probably in forestry too – the problem of adverse environmental impact of economic activity is better solved by government regulation and incentives than by discount rate prescriptions.

Conclusion

Declining discount rates should not be applied indiscriminately. The application of declining discount rates depends strongly on whether they are understood as a prediction of future rates of return to alternative investments or as a consequence of the increasing uncertainties of future rates of return.

In the former case, time-inconsistencies will occur as described above. The latter case, however, provides only a justification for applying declining discount rates in the "unreduced" Faustmann formula (2), not in the Ohlin condition (4). In this case, the suspected time-inconsistencies will not occur.

The case for predicting declining rates of return to capital in the distant future are considered to weak, but strong enough to include the possibility of such a scenario with some probability. Defining a range of scenarios with constant discount rates and probabilities and deriving the expected discount factor from it will probably adequately cover this scenario.

Applied along these lines, declining discount rates do not affect the optimal rotation period in itself and do not give rise to the suspected time-inconsistencies.

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