The economics of spacetime. An ecological perspective on social decision-making

by *Niels Peter Hahnemann* [1]

Abstract: Spacetime is a framework-independent perspective in coordinates spanned by space and time variables. The paper employs the framework, through the method of "mapification" of concepts discussed by mathematical category theory, to gain a perspective on the equilibrium of a sustainable economy. Economic sustainability is usually regarded as a state of disequilibrium. This limits the power of ecological systems to cause economic adjustment. The concept of time is the complicating factor in equilibrium. The idea of wristwatch time is discussed, which leads to a model of equilibrium with externalities illustrated by a discussion of population dynamics in an ecological system. The conclusion is that a social system can only be sustainable as an economy.

Keywords: sustainability, scalability, general equilibrium, relativity, category theory, economic ecology, JEL classification system B4.

Introduction

Ecological economics is the science about sustainability. It signifies a new development in economic theory towards research in the relationship between the economy and the external world. However, sustainability is on most definitions regarded as a normative notion about the way humans should act towards nature and towards one another and future generations.[2] The question then becomes: what is the challenge more in particular to economic theory from ecological principles? In fact, problems about how humans should act, or about the wellbeing of people, have for centuries been the key subject matter of economics. It is the subject of welfare theory. However, to address the relationship between the economy and the external world means, in effect, also to revisit the discussion of economic efficiency and equilibrium, now from the perspective of the carrying capacity of an economy. This is the focus of the present paper.

The problem is, of course, as old as economics. The classical economists had a model with limited and non-reproducible resources from land leading to decreasing average returns regardless of technology. It was a model of the non-sustainable consequences for the economy of an external environment. In neoclassical eco-

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[2] Stefan Baumgärtner, Martin Quaas (2010), What is sustainability economics?

nomics resources were regarded as unlimited and reproducible, but the economy was nevertheless limited by efficiency concerns, that is, by the problem of maintaining a fully exhaustive use of resources with respect to certain objectives. In ecological economics, finally, the concern about the reproducibility of resources and their limitations has returned, and with a vengeance. The focus is now on sustainability, that is, on a state where the external environment of human decision-making somehow has to be counted in in order to address the reproducibility of the economic system over time. The problem is that this state is usually not regarded as a state of equilibrium. Economic sustainability is rather being regarded as a state of disequilibrium, which limits the power of ecological systems to cause economic adjustment.

The purpose of the present paper is to produce an equilibrium concept of sustainability. The method employed is to update our visualisation of an abstract economy to the perception of a physical object in spacetime. By spacetime is meant a perspective spanned by an independent framework of space and time variables, the coordinates of which signify events. It is by realising the importance of comprehending the independence and therefore arbitrariness of the framework for decision-making with respect to decision that we may come to appreciate the ecological argument in economics.

The paper is organised as follows. The next section discusses the methodological framework for the analysis of decision-making in an economic programme with both space and time as variables. It employs a kind of philosophical algebra according to which concepts can be objectified as maps in a category. The subsequent section analyses the causality problem inherent in the economic programme and its justification through the concept of equilibrium. The approach to the transitivity problem is seen to be crucial in this respect. After this follows a section on the determination of time as the complicating factor in equilibrium, and the idea of wristwatch time is discussed, which leads to a model of equilibrium with externalities in the subsequent section. The next but final section illustrates the model by a discussion of population dynamics in an ecological system. The final section provides for the conclusion: a social system can only be sustainable as an economy, on which the economics of spacetime gives a perspective.

A "mapification" of concepts

Economic theory should ask: can decision-making be understood as the pursuit of finite objectives within some unbounded universe of knowledge not governed by superior design, that is, in a spacetime where space and time coordinates are simultaneously determined? If so, we would need observable "data" consisting of objects, maps, domains and rules, that is, a meta-set or system in mathematics called a category, and – as the system is supposed to be truly self-referential, or self-contained – we would need to determine the structure of causality that is decided in such a system.

The underlying idea is to employ a kind of philosophical algebra according to which concepts can be objectified as maps in a category. A category is a meta-set or system of "data" consisting of objects, maps, domains, and rules.[3] An object in a category is a domain or codomain of a map. Objects have so-called morphisms if the maps in a category observe identity and associative rules,[4] and the mapping of one category into another by a so-called functor can then be said to be structure preserving. A category is characterised by its morphisms, not its objects. The rule that a morphism observes is equal to its assignment, it cannot be considered independently from its assignment, or from what it accomplishes. This means that the rule is never empty; decision-making is a non-empty map. The map represents what the rule accomplishes.

Categories provide a "mapification" of concepts that enables a combination of concepts so that a composition of maps in a category can be analysed while preserving the composition's structural properties. In this manner, basic mechanisms at play within economic problems are brought to the fore. This approach is different from the "Bourbakist" approach to mathematical economics, which attempts a full axiomatisation of economic concepts regardless of contents.[5] In ecology, however, the emphasis is on the mechanisms of a self-contained and regenerating system.

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[3] The expressions "mapification of concepts" and "philosophical algebra" are from F. William Lawvere, Stephen H. Schanuel (1997), *Conceptual Mathematics. A first introduction to categories*, pp. 127 and 129. A definition of a category is given on p. 21. The following draws heavily on, in particular, pp. 3-7, 40-49, 68-80, 86-87, 101, and 213-224.

[4] Identity law: if the map f: A → B where A, B are objects then $1_B \circ f = f$ and $f \circ 1_A = f$ where 1_A , 1_B are identity maps of A, B and ∘ is a composition of maps. Associative law: if the maps f: A → B, g: B → C, h: C → D then $(h \circ g) \circ f = h \circ (g \circ f)$.

The economic justification of particular axioms is therefore important. We therefore need to address three questions generally posed to science: where (description), why (explanation), and how (prediction).[6]

Firstly: “where”? Economic theory is concerned with the purposeful action of human beings. There should therefore be no decision without causation, and causation necessitates simultaneity, that is, the cause must be conjoined with its effect. Decision-making would then work like motion mapped with space and time coordinates by a space and a time map, respectively. The space-map would describe a set of alternatives (exchanges) represented by some numbers, and the time-map would report the simultaneous positions of two hands on a clock. The coordinates of this system would locate events in economic spacetime, that is, in a system where space and time are welded together into a uniform continuum. Simultaneity then looses its absolute and objective meaning and becomes no more than an abstract notion, the material quality of which comes from a concern with the contact between two “bodies” that are events that occur at points separated by a distance in space and to which can be allocated a special time. The contact is designated as “spacelike” (spatial) in the manner of a decision giving cause to an effect that extends certain bodies according to some rule into space, and as “timelike” from the continuity and persistence of those bodies. The problem of simultaneity is then captured by a map from time to space that represents motion. Thus, motion is not simply considered like a track (like in time exposure) but more like a mapping procedure.

Secondly: “why”? Decision must be chosen over coercion like in a programme because the choice, being guided by purposefulness, objectifies causation in contrast to the case of coercion. In an economy one would want to decide x according to some rule f in order to get y. Economics therefore needs to consider a map such as the problem $y = f(x)$, where x is an object in a domain (like commodity space or a set of events). y is a unique solution to a problem, but it is a value that has to be computed, that is, it has to be estimated through approximation like in the maximisation of a utility function. The problem $y = f(x)$ would therefore need to be an-

[5] An approach to mathematical economics initiated from the mid-20th century by a group of mainly French economists.
[6] This is inspired by the discussion in Albert Einstein (1926), Space-Time.

swered by supply and demand maps that should delineate a space consisting of a plane and a line such that relative price values and some absolute price level (a macro aggregate or index) are determined. We can call this for the space problem. The economic programme problem would then consist of a combination of the space problem with the simultaneity (or time) problem based on the assumption that decision-making is reducible to two maps, one from time to a plane, and one from time to a line.

Numbers such as prices can represent a point on a line, a pair of numbers can represent a point in the plane, and three numbers can represent a point in space, *if* an origin (initial conditions, or endowments) and a unit of measure are chosen. This means that there is movement involved along perpendicular axes of a coordinate system even though movement seems to have disappeared from decision by being turned into a determination of values in a set of some real or rational numbers. We can write this phenomenon as:

(1) *space* = *supply* ∘ *demand*,

where *space*, *supply*, *demand* are maps and ∘ denotes the composition of maps such as *supply following demand*. The composition could, in turn, on certain assumptions be performed by some universal mapping properties such as multiplication or addition where multiplication would represent the operation "and", addition "or". Markets would accordingly, through supply and demand, work by a (basically non-linear) combination of maps that express independent inclusive decision-making, putting things together.

Thirdly: “how”? The explanatory power of economics depends on a given model's capacity to demonstrate equilibrium and, thus, to predict. Equilibrium is the reply given to a pre-scientific "how" question, but what would the structural properties of equilibrium be if we "mapify" the concept in a category and analyse equilibrium as a composition of maps? We have seen above that supply and demand works trough multiplication, or addition, of maps. The demonstration of equilibrium would then be the same as showing that the inverse to this process exists. The inverse of multiplication is division, and the inverse of addition is subtraction. Given the composition of a supply and demand map there would be a known object *D* in commodity space and a map *f* that ends up plotting the set of numbers through some "naming" onto an object *X* such as points on the line *L*, or the plane *P*. If this plot, through which elements of *X* are named by *D*, is invertible as in a division problem so that there to each object *X* is coordinated some numerical name into *D* such as the set of numbers, this would give information about *X* as well. Such an invertible preference map is called an isomorphism. A principal use of isomorphisms is coordinate systems. In general, economics would want to study markets characterised by the isomorphism $f: D \to D$. A map where the domain and the co-domain are the same object is an endomap, and according to L.E.J. Brouwer's fixed point theorem, which was presented in 1911,[7] a continuous endomap has a fixed point *x* for which $f(x) = x$, that is, from any *x* in a commodity space domain *D* you can have one *x* in commodity space codomain *X* by some operative procedure *f*. $f(x)$ would only be equal to *x* on the boundary of a domain. The theorem describes a sort of self-fulfilling process in the form of a mapping from a structured domain into itself.

Thus, there is a point x mapped into itself where the market problem is solved and equilibrium obtained. Invertibility is necessary for equilibrium, that is, for the existence of a solution. Commodity space needs to consist of a closed disk *D* mapped by *m* (for markets) to a set of prices *P*, and for each price P_N a map *p* (for projection) would assign a point back on the closed *D* such that the composition *f* of $p \circ m$ takes $D \to D$. There would for this *f* be a point *x* such that $f(x) = x$ where *f* is a composite map. The map *f* would be perfect to the extent that it would include a map of the map that has mapped commodity space. Equilibrium follows from invertibility because if *m* has an inverse m^{-1}, it has only one so that $p = m^{-1}$. Invertibility thus involves two maps, *m* and *p*.

The argument pursued so far is summarised in the following manner: economics poses a problem that can be spanned by supply and demand maps in terms of variables such as prices and quantities. I want to analyse this problem as part of a wider mathematical category problem that has space and time as variables so that the condition of causality is satisfied. In economics we solve problems that decide *x* according to some rule *f* in order to get $y = f(x)$. The solution is an equilibrium. In a

[7] L.E.J. Brouwer (1911), Über Abbildung von Mannigfaltigkeiten. I draw on the discussion in F. William Lawvere, Stephen H. Schanuel (1997), Conceptual Mathematics. A first introduction to categories, p. 120-132.

category, we consider space and time as domains, or objects of maps such as variables, denoted by $\boxed{\text{space}}$ and $\boxed{\text{time}}$ respectively, that are themselves mapped by a functor denoted by →. The economic programme could then be written as a composition of three functors:

(2) $\boxed{\text{time}} \rightarrow \boxed{\text{space}} \rightarrow \boxed{\text{line = price level quantity or aggregate}}$ and $\boxed{\text{plane = relative price}}$

that is, as a map from time to space, a map from space to a line, and a map from space to a plane. A morphism is a composition of maps such as: $\boxed{\text{space}}$ = *supply* ∘ *demand*, or *demand* ∘ *supply*, where *supply* and *demand* are maps. Thus, the problem posed by economics is from this perspective turned into a morphism that collapses the economic programme into two maps, one from time to a line (an aggregate or level quantity such as the price level), and one from time to a plane (a relative quantity such as values or relative prices).

The causality problem

The causality problem (2) inherent in the economic programme has not been entirely clear throughout the history of economic thought. The classical economists were mainly concerned with value and did not really consider economic theory as being concerned with an equilibrium problem between independent supply and demand mappings. In modern economics the preoccupation with an independent framework has mainly been addressed as a problem of economic methodology, while the assessment of causation has been regarded as an empirical matter.[8] Causation has not been discussed within an independent spacetime perspective with both space and time working as variables. "Mapification" of concepts in a category puts us, however, in a position to realise that the economic programme can be effectively collapsed into a macroeconomic and a value problem, or into aggregate and relative price determination, respectively, if programmed through time so that the mechanism of supply and demand involves causation and equilibrium consequently obtains.

Equilibrium is a situation, where commodity space D is mapped into itself as in:

(3) $D \rightarrow X \rightarrow D$ with $f\colon D \rightarrow X$ and $f^{-1}\colon X \rightarrow D$.

[8] See, in particular, Clive Granger (2004), Time Series Analysis, Cointegration, and Applications.

Thus, f is a preference map that plots real numbers through "naming" onto X such as a line L of aggregates or a plane P of relative prices. The existence of equilibrium means that the equations have a solution. If f is invertible, there exists an isomorphism $f^{-1} \circ f = f$ such that f: $D \rightarrow D$. f is then an endomap.[9] f is the economic programme. The mapping has a point contained in its image, a fixed point, which is then equilibrium.

The concept of equilibrium basically amounts to a justification of the causation in an abstract system. Every element in commodity space is adjusted in equilibrium according to a comparable and ordered preference function by the assignment of a number. Markets must be complete, and there must be time markets for all commodities allowing forward trades through some time preference, that is, markets are also contingent. Economic theory can then define such a system as a group, that is, as a set of axioms with an associative composition in all its elements. With prices given a combination of maps can be composed and an inverse found so that equilibrium exists.

I denote such a group ECON101. Its existence presupposes an assumption that the category of economic reasoning is capable of moving from morphisms to isomorphisms, that is, to invertible maps. Whether that is feasible depends on the composition of an economic programme map that, in the form of a decision problem, satisfies the causality condition of relation (2). It is here that mathematical topology has provided feasibility by the assumption of convex sets, which is tantamount to the exclusion of interdependencies and externalities and thus allows prices to be considered as given in a competitive equilibrium. ECON101 consists of the set of decision-making market problems. A problem is a decision problem if it requires the answer “yes” or “no”. The particular decision problem that we need to address is the transitivity condition, which is the cornerstone in all general equilibrium economics and without which the welfare theorems would not hold: if C is preferred to B, and B is preferred to A, then C is preferred to A.[10]

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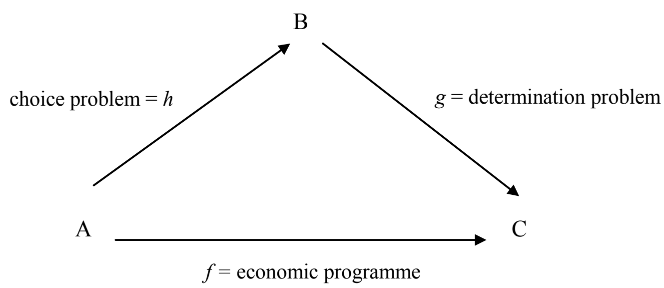
[9] This is Uzawa's equivalence theorem, that is, a proof of equivalence between a Walrasian equilibrium existence theorem and Brouwer's fixed point theorem, see chapter 6 in K. Vela Velupillai (2010), *Computable Foundations for Economics*.

[10] Transitivity follows from Axioms I and II in Kenneth J. Arrow (1963), *Social Choice and Individual Values*.

Basically, transitivity implies ordering according to consistent preferences. An economic programme considered from the perspective of causality would need to arrive by way of trading from *A* to *C*. Consider three domains or codomains *A*, *B*, *C* and three maps f: $A \rightarrow C$, h: $A \rightarrow B$, and g: $B \rightarrow C$ that together forms a "decision triangle". In decision problems we would know the outcome *C* (the thing we want) and the starting-point *A* (the thing we've got), and then going from *A* to *C* would be according to some rule *f* that stipulates that we need to proceed via some *B* ("yes"), otherwise we cannot get there ("no"). There is a restriction on *f* given by the need to have simultaneous causality in *B*. This is in fact a division problem that comes from inverting the transitivity condition. The problem is then whether such an inverse exists. If it exists transitivity holds, and decision-making can be made according to some truth concept. There would be no arbitrariness involved by going from *A* to *C*. You would have to proceed via *B*.

The transitivity problem can be approached using graph and category theory. Graphs are mathematical objects used to model pair-wise relations between objects. A graph is an object in a category, and categories are graphs where the relations between objects are compositions. *A*, *B*, *C* are vertices and *f*, *g*, *h* are arcs of a graph. A division problem could, in particular, be analysed as a detour graph. Causality means that decision needs to proceed via B in order to get to C; it cannot go to C directly but have to take the longest distance via B. Decision making becomes a recursive problem the solution to which depends upon the existence of an inverse to the composite map *f*. The division problem graph is illustrated in *chart 1*.[11]

Chart 1



The division problem is here seen to consist of two sub-problems.

[11] The chart is taken from F. William Lawvere, Stephen H. Schanuel (1997), *Conceptual Mathematics. A First Introduction to Categories*, p. 45-49 in the chapter on isomorphisms.

1: The determination problem is that we do not know how C has happened, that is, there is a B with one element so we know how to get from A to B by a map *h*, but not how to get from B to C by a map *g*, that is, we do not know *g*. If the problem has a solution *g* then $f = g \circ h$ is determined and therefore decided by *h* (“yes”), or: *f* is a function of *h*. We can say that, in determination, the composition $f = g \circ h$ leading from A to C via B is decided by *h*.

2: The choice problem is that we do not know how to get to C, that is, there is a B with more than one element and we do not know which one, that is, we do not know how to get from A to B by *h*, that is, we do not know *h*, but we know how to get from B to C by *g*. If the problem has a solution *h* then $f = g \circ h$ is chosen and therefore decided by *g* (“yes”), or: *f* is a function of *g*. We can say that, in choice, the composition $f = g \circ h$ leading from A to C via B is decided by *g*.

Thus, the composite economic programme map *f* is seen to embed a problem of either determination or choice. *f* can be decided by either *h* or *g*. But *f* can only be a composition of $g \circ h$, which means that there is lack of control because the reverse composition $h \circ g$ would not necessarily lead to *f*:

(4) $g \circ h \neq h \circ g$

The product $g \circ h$ does not necessarily equal the reverse $h \circ g$ because the choice problem is different from the determination problem. The difference is not zero but rather equal to a technological uncertainty factor:

(5) $\Phi = (g \circ h) - (h \circ g)$.

This equation captures what I call the economic decidability problem. From where or what does the problem originate? Φ is a consequence of the composition of maps that constitute the economic programme. The problem follows from the determination of space and time coordinates that is a consequence of the embedded decidability problem. Φ is zero if the economic programme is only a matter of supply and demand. If there is more to it than this it is because there, embedded in economic decision-making, is some technological constant Φ that captures objective uncertainty.

The composition of maps that constitute the economic programme problem would then need to involve this constant as a factor in decision-making:

(6) $time = \Phi * space$,

where *time* and *space* are maps and * denotes multiplication. Relation (6) captures the idea that there should be simultaneous determination of space and time coordinates. It is a definition of an equilibrium condition where Φ captures forces that are external to the model but has an effect on its equilibrium. Therefore, relation (6) describes equilibrium with externalities. It gives a definition of sustainable equilibrium because it describes a relation between space and time that comes with externalities.

The determination of time

If sustainability is a form of equilibrium the idea then is that this only gives meaning if it is non-efficient equilibrium. The relationship between space and time, which this idea involves, is however difficult to comprehend. But there seems no escaping the conclusion of the discussion above: the complicating factor in the equilibrium concept is time. The problem is then to examine how time is determined. Does that even make any sense? The following section examines the role of time in economic equilibrium.

How is time determined? According to relation (6) time would be a property of space called spacetime like in the measuring of seconds in meters as in (the inverse of) velocity. This is equivalent to, say, in physics where gravity is a force that expresses mass as distance using a gravitational constant (that actually is difficult to measure exactly). The gravity of economic decision-making would be rationality (which also is very difficult to measure with any precision). If Φ = 0 there is no simultaneous determination, and the time concept then refers to a one-dimensional frame that is not a property of space.

Measuring for example seconds in terms of meters reflects spacetime's bending, which as a side effect causes uncertainty. Economics would similarly want to determine Φ as it is a factor that bends rationality. The determination would be done by map-making. The existence of a constant in map-making does not mean that economic decision is confronted with uncertainty. It means that economic decision-making is producing uncertainty. Decision and uncertainty are complementary similarly to knowledge with respect to information. Production of uncertainty means that map-making can go wrong. Φ is as constant an approximation to a rule that assigns an element of a time-domain to an element of a space codomain. If we knew the value of Φ we could exactly discount future decisions, that is, determine objective uncertainty, which obviously is impossible. We would have determined a factor that could substitute perfectly for time preference. If, however, we can get an approximation of Φ we can improve our forecasts of economic reason.

The map-making of the economic programme depends on the constant Φ. Decision takes exchanges and trades from time to space, but there is a distortionary factor Φ that bends human mechanics. The effectiveness of the economic programme depends on an approximation to the hidden, or implicit, constant Φ, which captures the true gravity of economic decision-making, that is, rationality. Φ is a constant that reflects the rationality of human beings that we cannot explain. It is an outcome of uncertainty in economics. Φ was implicitly assumed to be equal to zero in Brouwer's argument. The question, which "mapification" allows to ask, is then: can an estimate of Φ be truly transformed into an isomorphism? We realise that an answer to this question cannot be given without an examination of, how time is determined.

The constant Φ expresses the balance between utility and risk, or risk anticipated and risk encountered, that has driven economic considerations ever since Daniel Bernouilli in 1738 argued for the need to equilibrate desires and satisfactions of human beings.[12] Φ reflects the need in economic theory for a law that demonstrates the gravity of the balance between utility and risk, or demonstrates why necessity is really necessary. The idea in the following is to discuss time as motion, like gravity. Time is seen as determined time, not given frame time, using the concept of wristwatch time taken from theoretical physics.

It is actually a fairly standard procedure, when dealing with preferences, to try to give a numerical description of an ordering such as a "greater than" relation by using some arithmetical transformation – only that the transformation in question in this case must concern time. How is time as an object for time preference determined? There is perhaps even nothing new about a time preference map – except that it is time that is to be determined in terms of space, thus providing for true

[12] Bernouilli, Daniel (1738), Exposition of a New Theory on the Measurement of Risk. The paper was republished in 1954 at the high point of mathematical Bourbakism in modern economics.

causality. How can time in economic problems be represented more specifically in terms of price and relative values? The idea is that if time is understood as the usual absolute frame time with a clock tick-tocking independently along, then decision-making as mapping is not invertible and there would not exist equilibrium, that is, there would follow no causality from rational considerations concerning human decision-making. If time, however, is transformed to true wristwatch time (for example by a map known as the Lorenz transformation) then the composite map of the economic programme would be an isomorphism. ECON101 would preserve its group properties; it would have an inverse. Without a proper transformation of time, no ECON101 group exists except for a model where externalities are the cause of disequilibrium and inefficiency as in the Arrow-Debreu model of general economic equilibrium and as maintained in the welfare theorems.

The intuition behind such a transformation, and the justification for the economic relevance of the concept of wristwatch time, is that decision can be regarded as having no mass, so decision could constitute an absolute and finite limit to determination and choice. There can be no decision about decisions in the framework of some absolute time. That would be a decision above decisions, which obviously is not a decision. Decision must be regarded as somehow bended in the space of decision-making, which then is turned into spacetime. The problem is, how "bending" can be understood?

Bending of decision-making reflects the uncertainty in economics exemplified by the choice or determination problem. Basically, it arises as soon as the division of labour, which the classical economists were the first to discuss, delays desires with respect to satisfactions making decision as opposed to coercion an issue. This is an effect of the market economy. Economic theory would attempt to measure the velocity that matches this delay by fitting utility to expectation thus settling the determination or choice problem through some trade. Such measuring is calculated from a point of view, and any viewpoint can be equally right if the possibility of being wrong is accounted for. The measuring delineates a universe from inside of rationality, which becomes a self-referential system with no reference to absolute space or time. Because rationality is thus discussed as purely self-referential, the possibility of being wrong, making errors, is included. Trade participants would then be on local wristwatch time. People would, so to say, be on the inside of a decision-making universe not allowed to look in from the outside.[13]

This universe would be self-contained, which however does not make it purely subjective. "Self-contained" simply means that the universe is unbounded, while its limitedness is in its finitude, that is, in its existence. Objectivity would still hold. According to the relativity principle discussed by theoretical physics, laws must be the same regardless of measurement, that is, irrespective of time and space. This principle must surely hold for any theory, which calls itself science. In economics we would be concerned with measuring a finite but unbounded universe of decision-making (like the surface of the earth), and map-making in such a universe would be needed as guide for economic decision, explaining causality and predicting events.

The relativity principle should secure that economic laws are the same no matter what measurement, that is, regardless of coordinate system applied. This is a difficult problem when time is involved, as pointed out by Albert Einstein in his theory of special relativity. The simultaneity involved in causality is an event that has its own speed indicated by the event of flashing light signals with a finite speed that coincides with a time signal on a watch. The understanding of causality then involves different and shifting coordinate systems. Any arbitrariness originates from the coordinate systems. How should the relationship between these different coordinate systems be? Where would truth be in all this? This is the subject of relativity theory. How can laws of motion for a particular event, established in one coordinate system, also be valid in a different system with a somehow shifted position relative to the first?

Causality requires simultaneity so that one event is causing another like one white billiard ball hitting a red one thus pushing it in a particular direction. Under-

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[13] The basic principle in assessing curved space, is that it restricts the capability of looking at a problem from the outside by determining the parameters of a single coordinate system. Rational judgement is rather constrained and conducted as by blind bugs living on a sphere: ”We define a curved space to be a space in which the geometry is not what we expect for a plane. The geometry of the bugs on the sphere or on the hot plate is the geometry of a curved space. The rules of Euclidean geometry fail. And it isn’t necessary to be able to lift yourself out of the plane in order to find out that the world that you live in is curved. It isn’t necessary to circumnavigate the globe in order to find out that it is a ball. You can find out that you live on a ball by laying out a square”, that is, by local measurements; Richard P. Feynman (1963, 1997), Six Not-So-Easy Pieces. Einstein’s Relativity, Symmetry, and Space-Time, p. 117-118.

standing causality would make us sure about a similar event the next time. The speed of the ball would result from transformation-by-addition of speeds using a composition of coordinate systems given by isomorphisms. In theoretical physics, simultaneity involves an assessment of time, which is relative to the speed of light flashes from observation that has an absolute and finite value. An appropriate theoretical transformation adding the velocity of light to the velocity of observation is therefore important for the physics of high speed phenomena and, thus, for theoretical physics. But the difficulty is that the velocity of light cannot be surpassed. How can the transformation-by-addition of speeds then be exercised? Only by making time a coordinate like other space coordinates, which implies that time becomes a property of space. The past, the present, and the future would exist simultaneously just like a car can be moved from one existing location to another existing location.

Is this of any importance to theoretical economics? The answer is: yes. Decisions are real world choices or determinations exercised in real time. The future and the past would from the point of relativity exist as simultaneous spacetime coordinates for human decision-making in as much as they are assigned a value. Obviously, human beings are not moving around at speeds even remotely resembling the speed of light, so the insights of relativity physics would seem to be irrelevant for a science like economics. However, does that also hold for the laws of decision-making? What is really the "mass" of economics: commodities, actions of people, decision-making? Decision has no mass so decision-making would constitute an absolute and finite limit for the economic world equivalently to the role of the speed of light in physics. There can be no decision above decisions, and that somehow bends the spacetime of decision-making, making future exist. Actually, it is not really controversial that the future exists in economics. It is usually represented by a price on the commodity of time such as the rate of interest or profit.

The relativity theory of Albert Einstein[14] pointed to the fundamental necessity of preserving the arbitrariness of different coordinate systems with respect to invariant laws of nature that are effective in their own right. We can safely consider it

[14] Firstly made public in 1905. The following is primarily based on the popular exposé in Albert Einstein (1916, 1952), Relativity: The Special and the General Theory. Note Einstein's modesty in the preface: "May the book bring some one a few happy hours of suggestive thought!" It surely did…

the provider of methodology and worldview of the science of sustainability. It is not a matter about some relativistic worldview. It is a matter about truth and the knowledge we can have of it. To give an example: the point is that if you choose to move from one point in space to another there must be an equi-proportionate choice involved moving from one point in time to another. This is the basic, fundamental idea. Time does not just pass; economic time is chosen. There is a difference between frame time and proper economic wristwatch time. Time is motion in economics. Wristwatch time is the time chosen or determined by the decision-maker when moving along the spacetime coordinates of choice. Frame time is different from the wristwatch time observed by different participants in an economic exchange. It is wristwatch time, not frame time that determines equilibrium as a simultaneous event that can be stable, and unique. Wristwatch time is objective, relativistic-rotated time:

I have decided that I want to move my body and, for example, take a walk in the borough of Østerbro, Copenhagen from here to there, let's say following a straight line route from Østerport Train Station to B93 Football and Tennis Club at Svanemøllen via Trianglen Square at constant speed (assuming that is possible). The coordinates of these three locations in Østerbro, Copenhagen would be known. Time of departure from Østerport (say 3 p.m.) would be known, and the event signified by a light flash. Time of arrival at B93 (say 5 p.m.) would also be known, and equally signified by a flash of light. What is my intermediate time when I pass through Trianglen? There is only one possible answer given my speed, that is, given that I need 2 hours for the full walk. Let us say that this time is 4 p.m., that is, I would pass through Trianglen at constant speed at 4 p.m. heading for arrival at B93 at 5 p.m. The 4 p.m. intermediate time is frame time. The point of the relativity principle is then that "proper time", equal to the time shown on my wristwatch when I pass through Trianglen, not necessarily is 4 p.m. (even when assuming that my watch is operating properly, does not loose time, and actually showed 3 p.m. when departing from Østerport). According to the relativity principle, "wristwatch time" at Trianglen would be the frame time (intermediate time) that maximises time from Østerport to B93 such that the relationship between the two times (frame time and wristwatch time) is constant at all points along the walk.

This wristwatch time would be determined. It would be a constant, which we in any real walks on the surface of planet earth at least safely can put at unity. Thus, we could assume frame time and wristwatch time to be identical and their relationship a constant equal to one. If that also were the case in economics, there would be no problem about objective uncertainty. The technological constant Φ that reflects the uncertainty from bended spacetime would be insignificant. But economics is not about walking, driving, or travelling in space. There is time in the economy to the same extent that economic theory wants to explain and predict, that is, to determine causality. We cannot put the constant Φ at zero in economic decision-making, the "speed" of which is not to be surpassed. Observers would note frame time, but the person moving and deciding would note wristwatch time. There should, therefore, be a determined and constant relationship between the two.

Equilibrium with externalities

If decision is motion and velocity is distance in terms of time, decision-making is a problem about multiplication of velocities carried through by a composition of maps. The decidability problem must then somehow be due to a problem of transformation-through-multiplication of different velocities such that their relationship is constant. Time is thus the key problem in the economic examination of causality. If time is reversible, then there is no causality. If time is irreversible, the value of the future is finite and cannot be zero. It will then be determined, we just do not know this value, and we cannot control it. But we can try to estimate it, getting a better and better approximation. In *Chart 1* that described the transitivity condition fundamental to mathematical economics, *A* comes before *C* but in the event they must be simultaneous in order for *A* to really cause *C*. There is a *B* in between. Concepts such as “action at a distance” and “the invisible hand” are not of much use in considerations regarding decision-making. There is no real meaning in the concept of simultaneity if it concerns distant events. Smith's invisible hand was only effective through his assumption about the importance of the "extent of the market" that caused decreasing returns to scale on average. He thereby laid the groundworks for avoiding the causality problem in 200 years of economic theory. But causality is simultaneous, and the question is: how can the limited period expressed by the simultaneity B be determined? It is in the determination of this simultaneity that time is given a spatial character.

What we want to do is to understand decision-making as motion implemented as a map from time to space. The important thing here is to keep motion invariant with respect to different time measurements. That can be done by an appropriate transformation, which observes the relativity principle. We just do not know how this appropriate transformation looks like more specifically. But if frame time and wristwatch time are not identical, the constant Φ cannot be insignificant. It will be different from zero, which means that markets can never be contingent. There will be incomplete markets. There cannot be missing markets if Φ = 0. True determined time, which we somehow cannot fully grasp, makes markets incomplete, which means that they will fail from time to time. To understand this requires an examination of the particular properties of Φ in order to see how it could be determined.

The complicating factor in the transitivity problem illustrated in *chart 1* was time. If the transitivity condition does not hold the fundamental welfare theorems do not hold. We can, furthermore, assume on the back of the discussion above that the existence of externalities and interdependencies in theoretical models is caused by the time problem. We must therefore reformulate the economic programme given by relation (2) so that it comprises externalities and interdependencies. That would lead to the problem:

(7) $y = f(x,z)$.

This programme decides x,z in order to obtain y by solving a problem according to some rule f. There are three variables:

x is given by a composition of supply and demand, that is, it is a space variable as shown in relation (1);

z is given by time, that is, $z = t$(time);

y is a quantity, for example a price, and it is given as a composition of a relative price or quantity plane and an aggregate price level.

In geometric terms, movements along any of the three axes x,y,z would be according to a transformation that relates one set of spacetime coordinates (x,y,z) to another (x',y',z'). The different coordinates then relate one coordinate system K to another K' moving with constant velocity relative to K according to the transformation, and we would represent this movement by schedules in a single coordinate system. The conventional way of analysing this problem can be described geometrically by a bounded two-dimensional surface *f* in a three-dimensional coordinate system, as in *chart 2*.

The surface *f* is spanned, for example, horizontally by the *x*,*y* space axes, while the *z* time axis moves vertically through the surface. Here, time is not transformed. At a given time point the surface is intersected once and only once. For each point in time the surface is sliced in two dimensions to obtain a single point (x_0,y_0,z_0). The timeline is shared. Moving in time shifts the horizontal surface vertically upwards. The result is a bounded surface in three dimensions, but the dimensionality of the problem is really not important, because the surface is basically a plane. Time is absolute like a clock tick-tocking along with no real past, present, and future. Relation (7) is approximated by the problem $y = f(x)$. Spacetime is not a problem in this kind of economics, while we discover that the dimensions to a problem, that is, its space or spacetime context has an independent effect on its properties.

Chart 2

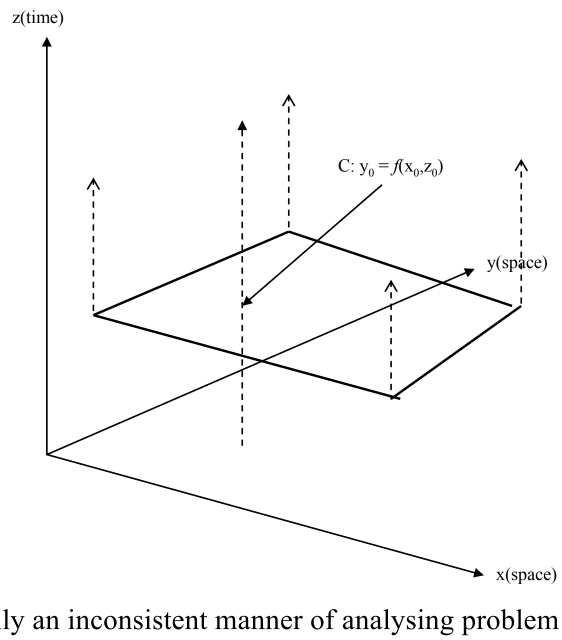

Chart 2 is really an inconsistent manner of analysing problem (7): $y = f(x,z)$. Causality should not be approached through shifting of a two-dimensional surface in three-dimensional space, where time is considered given and is not subject to a transformation. We saw in *chart 1* above that for causality to be effective space and

time must be contiguous, that is, there must be contact *B* between two "bodies" *A* and *C* so that each point (*x*,*y*,*z*) gets a timelike and a spacelike extension. I call this property "simultaneous causation". The points must be determined by independent coordinates of systems, which move uniformly to each other. At a given time point the surface *f* can then be intersected more than once without moving the *z*=*t* axis. The full solution *f* to problem (7) for all *x*,*y*,*z* is a timelike cone extended to a sphere in unbounded three-dimensional spacetime, shown in *chart 3*.

For every event in spacetime there is such a cone. The sphere *f* represents the solution to the problem (7): $y = f(x,z)$ for different *x*,*y*,*z*. Thus *z*(time) varies for each combination *x*,*y*,*z*. Otherwise there cannot be simultaneous causation. C is the present and efficient point in the full solution-set to the economic programme, but because we cannot decide on whether *f* is a composition of $g \circ h$ or of $h \circ g$ there is rather sustained a whole range of past, present, and future equilibria on the sphere.

Chart 3

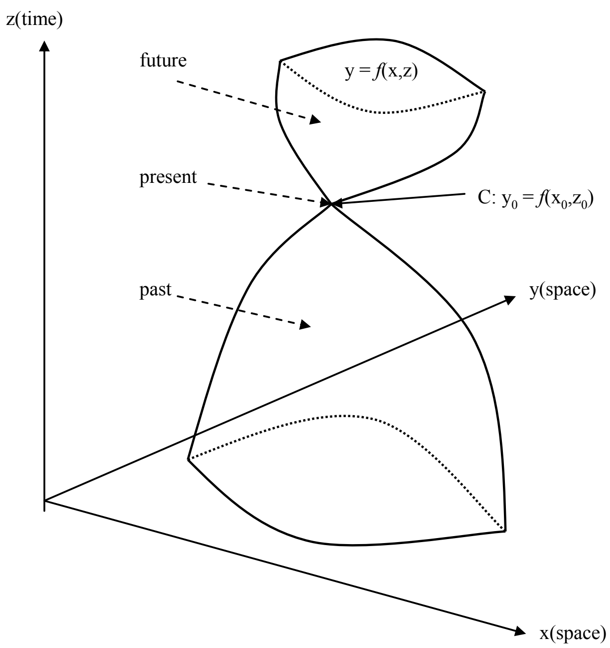

Chart 3 geometrically represents relation (6): *time* = Φ * *space*, that is, a relationship between time and space coordinates expressed by a constant Φ in non-Euclidean geometry. It illustrates sustainable equilibrium with externalities. Usually, points are represented by coordinate systems that move uniformly to each other,

thus enabling judgment of the same phenomenon in different systems of coordinates. This principle is extended to the time axis also in *chart 3*. The time line is not shared and the past, present, and future are therefore not dependent on the system of coordinates. Equal time planes are spheres spreading out from a certain point. The area spanned by the sphere f represents the solution to the economic programme. Through the chart we realise that this programme produces uncertainty because the past, present and future exist simultaneously. The equivalent to such uncertainty is incomplete markets and an economy that works according to ecological principles.

The presence of the constant Φ in any preference ordering expresses the problem of market making in a self-correcting sustainable economy. Time preference would concern all commodities. The intuition here is that market agents, that is, people due to the division of labour behind any commodity trade would not mind waiting somewhat regarding a particular trade. They therefore value the present differently than the future. The difference expresses the determined, limited, and finite character of time in economic causality. Markets trade a finite amount of resources, but they are expandable and therefore unbounded. This is basically due to their character of decision-making mechanism based on knowledge.

If $\Phi = 0$, there is full indeterminacy of time and no causality. Time is given as absolute frame time. If $\Phi \neq 0$ we have that *space* = *time* in some unspecified relationship, and economic explanation must then somehow attempt an estimate of the true value of Φ in order to really comprehend economic causality and improve on the predictive power of a model. Classical, modern mechanics operated with a relativity principle according to which all mechanical laws should be the same in any two coordinate systems. If (x,y,t) and (x',y',t') are points in two coordinate systems K and K', these points can be related by the Galileo transformation: $x' = x–vt$, $y' = y$, $t' = t$, where v denotes the distance in time between any two events.[15] An event is here a fixture in a coordinate space-and-time system, for example a displacement v along the x-axis. Any event localised on the x-axis would also be localised on the x'-axis, but there would be a velocity v, a distance with respect to time, by which

[15] I have here, for the sake of consistency with the three-dimensional economic argument pursued so far, omitted the fourth dimension of spacetime physics, and substituted z in *chart 2* and *3* by t.

the second coordinate system moved relative to the first. The Galileo transformation related one set of space-time coordinate systems (x,y,t) to another (x',y',t') moving with constant velocity v relative to the first. Simple addition of velocities would determine the overall position of a point in space.

The point is that time was absolute in classical mechanics, measured by clocks independently of position and condition of motion of the system of coordinates as seen in the Galileo transformation equation: $t' = t$. The transformation laws concerned space, not time. For space ex time the laws of nature were the same in any coordinate system. But you of course always had to measure the event, or state the law, in relation to some time. If this were not so you would have to find the one-and-only coordinate system, for example, the system absolutely at rest. This safeguarded the relativity principle. Time, however, was the same regardless of coordinate system according to the Galileo transformation. Time was "independent of the state of motion of the body of reference", as Einstein said.

Classical mechanics qualified time as an absolute measure, which the transformation did not alter. The event was fixed in time. Distance per time, that is, velocity would differ from one coordinate system to the next, but the law of propagation would be the same, for example that the movement in question was linear. However, Einstein pointed out through his very categorical reasoning regarding the relativity principle that this led to a contradiction. To escape this contradiction, you would have to submit time also to the principle of relativity; otherwise the principle could not hold for the velocity of light. As any object, light moves according to the equation: $x = ct$, that is, with a velocity c. The speed of light c is a physical quantity: the speed of the rays emitted by the sun. Light is moving very fast, but it has nevertheless a fixed speed. There would be no problem if light had unlimited speed. That is, however, not the case, as can be seen when time is measured by very precise and sensitive clocks.

With absolute time, two or more distant events are simultaneous, that is, happens at the same point in time if the displays of clocks, positioned at the place of the events, are the same. But measuring the degree of simultaneity and, therefore, time depends on the movement of light, which travels with a certain speed. An event is now not a fixture in space and time, it is a coincidence between signal and reception that captures simultaneity in spacetime.[16]

To safeguard the relativity principle that the distance per time c of light is invariant under any coordinate system, Einstein applied an extension of the Galileo transformation known as the Lorentz transformation. Einstein used the example of railroad cars moving vis-à-vis an embankment, and light flashing on both, in order to simulate two coordinate systems. He demonstrated that there was a contradiction in modern physics concerning the limiting velocity of light. Einstein solved the contradiction by using the Lorentz transformation. He started dealing with different coordinate systems and then transformed one into the other as required by the relativity principle.[17]

It was the limiting velocity of light, that is, a physical property, which necessitated the transformation of one coordinate system into another in order to safeguard the relativity principle. What Einstein did was to deduce a number of revolutionising consequences from first principles and thought experiments in order to safeguard science. An equivalent transformation is called for in economics. The Lorentz transformation, or some similar approach, is the missing link. The framework of absolute space is a problem for social science: it is impossible to justify, it jeopardises the scientific stature of economics, and it tends to disregard the ecological principles embedded in economic thought.

Economics as ecological population dynamics

Einstein rejected absolute space because of the absolute and finite limit to velocity of the velocity of light. Could that be considered as some kind of a paradigmatic opening also for social science, as science? Yes, it could. We have seen above that time is represented in terms of space in economic decision-making. Thus, must economics similarly reject absolute time because of the absolute and finite limit to reason of decision-making? Yes, it must. Would then the universe of

[16] ”If there is no cause and effect link and no material body concerned, then we may well have something travelling faster than light”; Herman Bondi (1962, 1964), Relativity and Common Sense. A New Approach to Einstein, p. 145. My presentation draws heavily on Abraham Pais (1982), ‘Subtle is the Lord’. The Science and the Life of Albert Einstein, pp. 133, 139, 148, and 167.

economic spacetime be finite but unbounded, as in theoretical physics?[18] Yes, it would. It is a general principle that not necessarily has to do with the speed of light.[19]

Real-time decision-making with causality necessitates a supplementary transformation between frame time and wristwatch time keeping their relationship constant. With $\Phi \neq 0$ there is not full indeterminacy between time and space. Economic theory that maps in frame time only would be perturbed by an implicit Φ assuming that it truly has some value different from zero. A map in frame time would not be invertible and it would not reflect the true model. In commodity space consisting of a closed disk D, the map f of p following m that takes $D \rightarrow D$ such that $f(x) = x$ is not invertible if m does not have an inverse. There is not one and only one $p = m^{-1}$ if there is full indeterminacy between time and space. The map from time to real or rational numbers cannot be decided. There comes a decidability problem.

However, with a proper transformation of time we are able to realise that markets are sustainable but not necessarily efficient. Unsustainable states originate from non-market forces that may upset the economy. The economic universe is limited to a finite but unbounded sphere and in this manner turned into an ecological problem about the management of feedback mechanisms through a proper time transformation. An economy based on completely deterministic non-stochastic variables could through loop mechanisms generate random fluctuations. And when

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[17] “For experiments on the earth tell us nothing of the fact that we are moving around the sun with a velocity of approximately 30 kilometres a second”; A. Einstein (1922, 1953), The Meaning of Relativity, p. 24.

[18] “We imagine the surface of a large globe and a quantity of small paper discs, all of the same size. We place one of the discs anywhere on the surface of the globe. If we move the disc about, anywhere we like, on the surface of the globe, we do not come upon a limit or boundary anywhere on the journey. Therefore we say that the spherical surface of the globe is an unbounded continuum. Moreover, the spherical surface is a finite continuum. For if we stick the paper discs on the globe, so that no disc overlaps another, the surface of the globe will finally become so full that there is no room for another disc. This simply means that the spherical surface of the globe is finite in relation to the paper discs. (…) The disposition of the discs in the manner indicated, without interruption, is not possible, as it should be possible by Euclidean geometry of the plane surface. In this way creatures, which cannot leave the spherical surface, and cannot even peep out from the spherical surface into three-dimensional space, might discover, merely by experimenting with discs, that their two-dimensional “space” is not Euclidean, but spherical space”, that is, curved; A. Einstein (1921), Geometry and Experience, p. 48-50. Human beings in a social environment might be such “creatures”!

[19] N. David Mermin (2005), *It’s About Time. Understanding Einstein’s Relativity*, p. xi: ”The peculiarity of motion at the speed of light is just a special case of a more general peculiarity of all motion. (…) Many tricks with light that seem to give it a fundamental role in establishing the nature of time can, in fact, be done just as well using any other uniformly moving things to signal from one place to another”. In our framework, decision is motion.

time is determined time like any other variable transformation-by-addition of velocities requires a relativistic rotation of coordinate systems. The determined variables then somehow escape full control and instead become self-generated through a feedback mechanism with unavoidable uncertainty associated.

The best description of such an economy is given by population dynamics with direct applicability to sustainability and ecological economics. Scalability refers to the development of a population, that is, of a variable that impinges upon itself causing growth as in a dynamic system. If an event, or object, or mapping is scalable, it means that it is scale invariant, scale free, or self-similar. Non-invertible mappings are non-scalable, that is, they are tied to a certain or characteristic distribution (or scale), they do not observe a power law. They are not self-similar, not affected by uncertainty, or uncertainty can for all practical purposes at least be ignored except as a deficiency in knowledge about the real world. The pattern of such mappings is dependent upon certain conditions. There is not a self-organised stationary state in non-invertible mappings.

In mathematical terms: if a production function is concave, that is, if it has got increasing costs to scale and therefore decreasing returns, then the complete prediction of the economy from $t = 0$ to $t = \infty$, or the economic programme, is convex. The economic programme can then be linear, that is, observe the preservation of norm.[20] However, with externalities the structure preserving properties of such a model does not necessarily qualify as an isomorphism, and it does therefore not necessarily have an inverse. The economics of frametime thus needs convexity of preferences (the economic programme) in order to assume continuity, which is a necessary condition for the existence of an inverse (the problem map), that is, for equilibrium. With decreasing returns the economic programme is non-scalable and prediction symmetric with explanation. They can only differ with respect to the particular point on the *z*-axis, the frametime (see *chart 2*), from which an observation is made. It amounts to the Gaussian bell curve hypothesis of a density function

[20] As in the Hahn-Banach theorem, which is one of the four pillars of the “citadel” of mathematical economics, the other three being the axiom of choice, Bolzano-Weierstrass (boundedness), and the exclusion of the middle (existence of a zero value of a function). See chapter 1 in K. Vela Velupillai (2010), *Computable Foundations for Economics*.

where errors are random and mistakes cancel out. Market failure would be due to externalities.

On the other hand, if $\Phi \neq 0$ then the isomorphism studied by economics, f: D → D, should be seen as an event in spacetime. Each event is here naturally assigned a time, but there would be no natural assignment of a spatial location in one specific location time – a reflection of the finite but unbounded character of economic decision-making. It describes a more general non-convex eco-system with externalities.[21] It would amount to a non-concave production function that is scalable, that is, has decreasing costs to scale, which then causes increasing returns on the supply side and income effects on the demand side. This state is described in economic theory as imperfect competition with oligopoly, monopoly, or monopsony. Externalities are counted in as economic mechanisms that preserve sustainability. The math easily gets very complicated.

However, the feedback and loop mechanisms of this kind of non-linear approach would not reflect rational optimising behaviour. The mechanisms would not necessarily have an interpretation, or explanation. They would merely reflect a non-signifying, arbitrary, and thus basically natural system. Feedback would be sign of an eco-economy that is. There would not necessarily be optimising rational behaviour behind aggregate developments. The description of such a sustainable system was firstly given by Richard Goodwin, who in 1967 developed a model of economic fluctuations based on the predator-prey relation in Lotka-Volterra's model of population dynamics.[22] Similar models have been made of ecological systems by, in particular, C. S. Holling, who in 1973 described the resilience of population systems with low stability and large fluctuations.[23]

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[21] A good summary of these problems is given in Partha Dasgupta, Karl-Göran Mähler (2003), *The Economics of Non-Convex Ecosystems: Introduction*.
[22] Richard Goodwin (1967), *A growth cycle*.
[23] "The more homogenous the environment in space and time, the more likely is the system to have low fluctuations and low resilience. (…) The very approach, therefore, that assures a stable maximum sustained yield of a renewable resource might so change these deterministic conditions that the resilience is lost or reduced so that a chance and rare event that previously could be absorbed can trigger a

Conclusion

Classical economics did not describe a sustainable economy. Their theory gave a sketch of a model that attempted to analyse the regeneration of a natural resource, such as the population of human beings, as a production process dependent on the own resources of the population only. Regeneration depended upon the human population input, not upon other resources. Capital and labour were put in land. The output was corn or wheat that regenerated labour and in turn capital, which only counted as saved labour input. But the capacity in food production was limited, and there was a big externality in the biological growth capacity of the human population, which surpassed the growth of food production. The classicals had decreasing average returns to scale regardless of technological improvements. This meant that the returns per population unit were believed to decline in the long-term. The resulting growth function described a non-scalable economic ecology leading to a stationary state, that is, a physically non-growing economy.

Neoclassical and modern economics have basically maintained this approach due to the assumption of a convex economic programme with a concave production function. Disequilibrium and market failure are due to externalities. Georgescu-Roegen tried in 1971 to reintroduce the Malthusian view through a reinterpretation of the law of decreasing returns. He did this by applying the second law of thermodynamics. Entropy would increase due to the fact that spent work like energy is transferred to the environment as waste (pollution) with no more work being able to be extracted from it. He thus reiterated the classical idea about a natural limit to growth. Georgescu-Roegen thereby in fact disregarded the capacity of the economy to regenerate resources such as labour inputs thereby maintaining the human population.[24] It was in fact the very idea of a sustainable economy that was discarded.

The present paper has instead argued that a social system can only be sustainable as an economy, that is, as a system in which it is recognised that the evolution or growth of one population or resource has a causal influence[25] – an externality – on the evolution or growth of another, or of several others. Whether that influence

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sudden and dramatic change and loss of structural integrity of the system": C. S. Holling (1973), *Resilience and Stability of Ecological Systems*, pp. 18, 21.

[24] Nicholas Georgescu-Roegen (1971), *The Entropy Law and the Economic Process*.

is beneficial or detrimental, predator or prey, parasitic or cooperative is a matter for welfare economics to consider. The human population is regenerated through exchanges and trades, through consumption and production. It is in this respect that the economics of spacetime provides a sustainable perspective on social decision-making.

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[25] The idea of mutual causal influence between populations is discussed in Giorgos Kallis, Richard B. Norgaard (2010), *Coevolutionary ecological economics*.

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